Designing Charging Infrastructure for a Fleet of Electric Vehicles Operating in Large Urban Areas

Michal Koháni1, Peter Czimmermann1, Michal Váňa1, Matej Cebecauer2 and Šuboš Buzna1,3

1Department of Mathematical Methods and Operations Research, University of Žilina, Univerzitná 8215/1, SK-01026 Žilina, Slovakia
2Department of Transport Science, KTH Royal Institute of Technology, Teknikringen 10, SE-100 44 Stockholm, Sweden
3ERA chair for Intelligent Transport Systems, University of Žilina, Univerzitná 8215/1, SK-01026 Žilina, Slovakia

{michal.kohani, peter.czimmermann, lubos.buzna}@fri.uniza.sk, {michal.vana, matej.cebecauer}@gmail.com

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Abstract: Here, we propose a method to design a charging infrastructure for a fleet of electric vehicles such as a fleet of taxicabs, fleet of vans used in the city logistics or a fleet of shared vehicles, operating in large urban areas. Design of a charging infrastructure includes decisions about charging stations location and number of charging points at each station. It is assumed that the fleet is originally composed of vehicles equipped with an internal combustion engine, however, the operator is wishing to replace them with fully electric vehicles. To avoid an interaction with other electric vehicles it is required to design a private network of charging stations that will be specifically adapted to the operation of a fleet. It is often possible to use GPS traces of vehicles characterizing actual travel patterns of individual vehicles. First, to derive a suitable set of candidate locations from GPS data, we propose a practical procedure where the outcomes can be simply controlled by setting few parameter values. Second, we formulate a mathematical model that combines location and scheduling decisions to ensure that requirements of vehicles can be satisfied. We validate the applicability of our approach by applying it to the data characterizing a large taxicab fleet operating in the city of Stockholm. Our results indicate that this approach can be used to estimate the minimal requirements to set up the charging infrastructure.

1 INTRODUCTION

Road transport produces 20% of total carbon dioxide (CO2) emissions, which is the main greenhouse gas. While these emissions decreased by 3.3% in 2012, they are still 20.5% higher than in 2011 and it could have been even more if there is no economic crises (European Commission, 2015). It is therefore expected that in order to reduce CO2 emission in densely populated urban areas, it will be desired to continue electrification of individual and public transport. However, electrification of transport itself does not ensure the reduction of CO2 emission, also the higher penetration of renewable sources of electrical energy is necessary. Advances in battery technologies and continuously decreasing prices of electric vehicles may soon increase the interest in converting large fleets of vehicles serving urban areas into electric, because the expected benefits could be considerable due to high utilization of such vehicles. Thus, high purchase costs of a new electric vehicle can be more easily compensated by lower operational costs. To avoid delays in charging, caused by interaction with other electric vehicles, a choice of a fleet operator can be to build their own charging infrastructure.

The topic of planning charging infrastructure for electric vehicles is rapidly growing in the scientific literature. The attention of researchers has been focusing on creating models that would be able to predict the future expansion of electric vehicles (Sears et al., 2014) as well as models designed to estimate the size of the future demand for charging vehicles (Yi and Bauer, 2014). An approach, where GPS traces of vehicles collected in two Italian cities were used to extract the travel behaviour and to estimate the expected demand for charging vehicles, was used in (Paffumi et al., 2015; Gennaro et al., 2015). Such analysis can provide valuable hints when searching for suitable positions of charging stations. Data driven approach to predict the penetration of EVs in the region of Lisbon and the future refuelling demand was proposed in (Frade et al., 2011).

Optimization algorithms have been often used to address this problem as well. In (Dickerman and Har-
rison, 2010) was used a city transportation model to verify various locations of charging stations, which were generated by the genetic algorithm. A bi-level approach was proposed by (Jung et al., 2014). On the upper level is solved the location problem (considering the capacity of charging stations), while the total costs and waiting time are minimized. On the lower level is used simulation approach to evaluate each design. Simulation as a validation tool has been used relatively often (Sweda and Klabjan, 2011). In (Xi et al., 2013) authors developed the simulation-optimization approach, where the area is divided into regions. The OD-matrix for the regions is known and it is used to estimate the EV flows between them. Linear IP model is used to determine the location and size of charging stations subject to limited budget. Simulation model is used to estimate the expected number of vehicles successfully charged at each candidate location. (Dong et al., 2014) proposed an approach that allows for analyzing the impact of public charging infrastructure deployment on increasing electric miles travelled. A genetic algorithm is used to find locations of charging stations and it is evaluated by the activity-based assessment method. Combination of a simulation approach with a genetic algorithm that utilizes GPS traces of vehicles was presented in (Tu et al., 2015).

Several authors considered a location problem leading to a mixed integer programming problem. For example, in (Chen et al., 2013) the demand for charging electric vehicles on public parking lots was estimated, based on a traffic survey conducted in the city of Seattle. The suitable location of charging stations was found by minimising the costs and the optimisation problem was solved by a general purpose optimisation solver. A similar methodology was also used for the city of Lyon (Baouche et al., 2014) and the city of Coimbra (Cavadas et al., 2014). When designing a network of charging stations, capacity constraints are typically included in the model (Lam et al., 2014; Ghamami et al., 2016) and apart from minimising costs, the area covered within the driving distance is maximised (Yi and Bauer, 2014). Methodologically different approach has been presented in (Momtazpour et al., 2012). Here authors considered constraints implied by the daily activity of car users and the capacity of electrical network and the location of charging stations were found by the clustering algorithm. The advantage of this approach is that several types of constraints can be taken into account simultaneously, without significantly affecting the computational complexity of the algorithm.

The special class of models was developed to cover trajectories of vehicles (MirHassani and Ebrazi, 2013; Capar et al., 2013). This approach is applicable in the design of the charging infrastructure along a highway to cover for long distance trips. An elegant way how to locate charging stations along the paths was proposed by (MirHassani and Ebrazi, 2013). The approach is based on adding artificial links to the network graph connecting places that fulfil some reachability rules. The model locates the minimum number of refuelling stations along paths to make the path traversal feasible. The approach (Chung and Kwon, 2015) further extends (MirHassani and Ebrazi, 2013) by considering multi-period case.

As our contribution, we propose an approach that is purely based on an optimization approach where we combine location and scheduling problems and thus we avoid the need to validate the locations and capacity of charging stations by computer simulations. Because the approach is based on historic data it can be used to estimate the minimum design of the system that is sufficient to cope with various scenarios occurring in the past.

The paper is organized as follow: in section 2 we describe the data requirements and the methodology. In section 3, we describe the data used in the case study and we introduce the results of numerical experiments. To conclude, we summarise our main findings in section 4.

2 METHODOLOGY

2.1 Data Requirements

The proposed method to design the network of private charging stations relies on two datasets. The first dataset is expected to contain historical low-frequency GPS data describing the mobility patterns of individual vehicles that belong to the fleet. Data should be collected for several, typical and sufficiently long time periods representing relevant scenarios that should be included in the design of the charging network. To collect low frequency data is much easier in practice as there is no need to use expensive GPS trackers, however, such data are not precise enough to determine the travel distances. Therefore we need the graph model of the road network including data about nodes, edges and their elevation. Using this data we can perform the map matching and estimate the travel distances much more precisely by inducing them from the road network.
2.2 Algorithm to Determine a Candidate Set of Charging Station Locations

In this paper, we will explore the case where to recharge an electric vehicle, we use only the time when it is parking for a long enough time. Thus, if possible, we do not wish to affect the current trajectories that are taken by vehicle drivers. The proposed methodology allows then to evaluate what percentage of vehicles could be transformed to electric vehicles without affecting their operation with the minimal requirements on building the charging infrastructure. Therefore, we use the historic GPS data to identify the set of suitable candidate locations for charging stations. Here, we aim to identify locations where the large number of vehicles frequently parks. To do so, we proposed the following two-phase procedure:

- Phase 1: Identify the set of candidate locations for charging stations as locations where many vehicles tend to park for a long enough time.
- Phase 2: Identify the set of vehicles that can be served by selected set of candidate locations.

To determine the set of candidate locations, following parameters are used. As an parking event we identify the time period, when the average speed of a vehicle is below the maximum speed limit $V_{\text{max}}$ for at least a time period of $T_{\text{min}}$. To ensure that we select a relevant set of candidate location we require for each candidate to be associated with at least $M_{\text{min}}$ parking events taking place in its circular neighbourhood defined by the radius of $R_{\text{max}}$.

In the first phase, we process one-by-one the GPS traces of all vehicles executing the following steps (see Figure 1):

- Step 1: Identify in the GPS trace the traversals that have the average speed below the speed limit $V_{\text{max}}$.
- Step 2: Identify in the GPS trace the maximum connected sequences of traversals longer than the time period $T_{\text{min}}$.
- Step 3: Identify as a candidate location the last node of each connected sequence if there is no other candidate location within the distance $R_{\text{max}}$.

After processing all GPS traces we remove all candidate locations that are associated with less than $M_{\text{min}}$ parking events.

The second phase of the procedure evaluates the proposed set of candidate locations and identifies the set of vehicles that could be replaced by the electric vehicles. For each vehicle we evaluate its trajectory and we evaluate whether it could be sufficiently recharged during parking events, to cover the travel distances. Here we consider the unlimited capacity of charging points being locating in each candidate location. We assume that the capacity of each vehicle is $K$ (measured in kilometres), i.e. it corresponds to the reachable driving distance. As a vehicle is driven its state of charge is decreasing by substricting from it the travel distance. Each time unit when the vehicle is charged we increase its state of charge by the value of $P$. We record the number of vehicles that cannot be served by a given set of candidate locations. In the mathematical model we consider only vehicles that can be recharged, otherwise the proposed mathematical model has no feasible solution.

2.3 Mathematical Model

We aim to minimize the costs that are required to set up the charging infrastructure. Due to the fact that in our approach we expect to locate charging stations of the same type, we minimize just the number of charging points. Previous studies (Sweda and Klabaj, 2011; Xi et al., 2013; Dong et al., 2014) indicated that an important requirement is to consider queueing behaviour of vehicles while charging. Therefore we formulate a location optimization problem considering the scheduling problem to ensure that there exist a feasible schedule how to recharge vehicles.

We assume that the algorithm introduced in the previous section was used to produce the set of candidate location where it is possible to locate the charging infrastructure $I$. To schedule the individual charging time slots we split the time into the set of non-overlapping time intervals $T$. Then for each vehicle we distinguish two possible states (see Figure 2). A vehicle is either parking at the candidate locations and it is available for charging or it is located somewhere else where it cannot be charged.

The maximum number of charging stations is limited to $p$. We consider that the fleet is composed of the set of vehicles $C$ and each vehicle is equipped by the battery, which when fully charged allows for driving the vehicle for the distance $K$. From the data we
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3 NUMERICAL EXPERIMENTS

3.1 Data

In the case study we consider a fleet of more than 1,500 taxicabs operating in the area of Stockholm district, in Sweden. Each vehicle reported on average every 90 seconds its id, GPS position, time-stamp and information whether it is hired or not. For the case study we selected four weeks (see Figure 4), altogether comprised of 8,989,143 probe data records. These four weeks represent different scenarios: week 1 is a typical spring week with 1542 taxicabs, week 2 represents typical summer week with 1526 taxicabs, week 3 is the Christmas week with 1491 taxicabs and week 4 is a special week, when the major disruption of the public transport occurred due to many failed railway connections, with 1550 taxicabs.

As the reporting frequency of probe data is relatively low, to be able to measure the travel distances more accurately we map-matched the probe data onto the road network using the methodology proposed in (Rahmani and Koutsopoulos, 2013). Thus, we use the road network to estimate the travel distance. In the digital model of the road network each link is attributed a number of parameters, including the length, presence of a traffic signal, road class, speed limit and etc. The graph of road network used in the case study consists of 231,839 links. In Figure 5 we show accumulated number of traversals by taxicabs over all four weeks for all links of the road network. We have no traversals for 97,720 links, where 96% of these are class 5 links. Conversely, the largest frequency of usage we observe on class 1 links connecting the city of Stockholm with the airports and in the city centre.

3.2 Numerical Results

We set the following values of parameters: the driving range of all vehicles $K = 300$ km, the initial fraction of the driving range $\alpha = 0.5$, the charging speed $P = 5$ km/min., we do not limit the number of charging stations, i.e., $p = |I|$, $V_{\text{max}} = 0.1$ m/s and $T_{\text{min}} = 15$ min. When constructing the mathematical model we

extract for each vehicle $c \in C$ an ordered sequence of parking events $R_c$ and we determine the list $N_{cr}$ of all time intervals $t \in T$ that have an overlap with parking event $r \in R_c$. The fraction of the time interval $t \in T$ the vehicle $c \in C$ is parking we denote as $a_{cr} \in [0,1]$. To simplify the description of the mathematical model we define $B_{ac} \in \{0,1\}$, where $B_{ac} = 1$ if the vehicle $c \in C$ parks at the location $i \in I$ during the time interval $t \in T$ and $B_{ac} = 0$ otherwise. We use the graph of the road network to extract the information about the real travel distances. Vehicle $c \in C$ drives $u_r$ kilometres while driving from the parking event $r = 0$ to the parking event $r_c$. Each vehicle enters into the model with an fictive parking event $r = 0$ of zero duration and exits with an fictive parking event $r = r_c$ of zero duration. Symbol $M$ denotes the big $M$ constant.

Decisions are described by the set of variables:

- $y_i \in \{0,1\}$ for $i \in I$, where $y_i = 1$ if the charging station is located at the candidate location $i$ and $y_i = 0$ otherwise,
- $s_i \in Z^+$ for $i \in I$, representing the number of charging points allocated to the station $i \in I$,
- $x_{cr} \in \{0,1\}$ for $c \in C, t \in T$, where $x_{cr} = 1$ when vehicle $c \in C$ is being charged during the time interval $t \in T$ and $x_{cr} = 0$ otherwise, and
- $d_{cr} \geq 0$ for $c \in C, r \in R_c \cup \{0\} \cup \{r_c\}$ corresponds to the distance that the vehicle $c \in C$ at the beginning of the parking event $r \in R_c$ is able to drive.

Making use of these notation we formulate the location-scheduling problem that is shown in Figure 3.

In the objective function of the problem (1) we minimize the number of located charging points. Constraint (2) ensures that we do not locate more than $p$ charging stations. This constraints approximates the limitations that are associated with the establishment of a new charging station. Constraints (3) make sure that we can assign charging points only to located charging stations. In each time interval we cannot use more charging points than available as it is specified by the set of constraints (4). We initialize the system by limiting the driving distance to $\alpha K$, where $\alpha > 0$ is parameter of the model (see constraints (5)). Constraints (6) ensure that battery capacity is not exceeded and constraints (7) ensure contiguity in charging and discharging of batteries.
**Model formulation**

Minimize \( \sum_{i \in I} s_i \)  

subject to \( \sum_{i \in I} y_i \leq p \)  

\( M y_i \geq s_i \) for \( i \in I \)  

\( \sum_{c \in C} B_{ct} x_{ct} \leq s_i \) for \( i \in I, t \in T \)  

d\( ct \) \( \leq \alpha K \)  

d\( cr \) \( + \sum_{t \in N c, r} a_{ct} x_{ct} P \leq K \) for \( c \in C, r \in R_c \cup \{ r_c \} \)  

d\( cr \) \( \leq d_{c,r-1} - u_{cr} + \sum_{t \in N_{c,r-1}} a_{ct} x_{ct} P \) for \( c \in C, r \in R_c \cup \{ r_c \} \)

(1)

(2)

(3)

(4)

(5)

(6)

(7)

Figure 3: Mathematical formulation of the location-scheduling optimization problem.

Figure 4: The travel distance, the travel distance when the taxicabs were hired, the number of unique taxicabs and dates of individual weekdays in weeks 1-4 that we selected for the case study.

discretize the time in steps of 15 minutes. Results of experiments are shown in Tables 1 - 4.

Numerical experiments were performed on the computer equipped with CPU Intel (R) Core i7-5500U CPU with two 3 GHz cores and with 8 GB RAM. Mathematical model was solved using IP-solver FICO Xpress IVE 7.3.

Based on the initial experiments we selected the following values of input parameters \( R_{max} \in \{100, 500, 1000\} \) meters and \( M_{min} \in \{100, 150, 800\} \) parking events to cover the broad range of situations. In tables we report the following output values obtained from the algorithm that is used to determine a set of candidate locations: \(|I|\) is the cardinality of the set of candidate locations identified in the Phase 1, \( Cars \) is the number of taxicabs that are determined in the Phase 2 as the vehicles that can be served by the set of candidate locations. Optimization outputs are the following: \( Stations \) is the number of located charging stations, \( CP_{total} \) represents the total number of charging points in all charging stations and \( CP_{max} \) is the maximal number of charging points located in one charging station. Column \( Time \) contains computational time in seconds. We restricted the running time of a single experiment to 30 minutes. If the optimal solution was not found within this time limit, in the column \( Gap \) we report the relative gap between the upper and lower bounds of the optimal solution. In two cases the solver was not able to find any feasible integer solution within the given time limit what is indicated by the symbols ‘***’ in the particular row of the table.

To verify the proposed approach, we visualised the resulting locations of charging stations for the scenario Week 1 and the parameter values \( R_{max} = 100 \) meters and \( M_{min} = 150 \) parking events. Results are shown in the Figure 6.

### 4 EVALUATION OF RESULTS AND CONCLUSIONS

In this contribution we present an initial design of the method to deploy the charging infrastructure for a fleet of electric vehicles operating in large urban areas. The operation of the fleet is described by the GPS traces characterizing the actual travel patterns of
### Table 1: Results of numerical experiments for the scenario Week 1.

<table>
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<th>$C_{\text{max}}$</th>
<th>Time [s]</th>
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### Table 3: Results of numerical experiments for the scenario Week 3.

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Individual vehicles. In the first phase we used a practical procedure to derive from data a suitable set of candidate locations for charging stations, where the outcomes can be controlled by setting a few parameter values only. In the next step, vehicles that can be served from the set of candidate locations are selected. In the second phase, we formulated a mathematical model that combines location decisions with...
scheduling decisions to ensure that for a given design there exists a time schedule that allows satisfying requirements of all vehicles selected in the first phase. The limits of the proposed approach were tested by applying it to the real-world data characterizing the driving behaviour of a large taxicab fleet operating in the city of Stockholm. From the numerical experiments we derive the following main conclusions:

- Our results indicate that this approach can be used to estimate the minimal requirements to set up the charging infrastructure. The proposed method is able to handle relatively large instances of problems independently on the scenario. Problems with \( R_{\text{max}} \in \{100, 500\} \) and \( M_{\text{min}} \in \{150, 800\} \) are often solved to optimality or with small gap only.
- Charging points are typically located at parking lots in the vicinity of airports, railways stations and other public spaces, which seem to be natural locations for them.
- When comparing the results across selected scenarios we find similar numbers of located stations in weeks 1, 2 and 4 and significantly smaller number of charging points in week 3, which is the most quiet week.
- We did not limit the number of charging stations by setting the value of the parameter \( p \). Hence, the number of charging stations was limited only by the set of candidate locations \( |I| \). From the
solutions we can see that if $|I|$ is large enough, the optimization model has the tendency to select the large set of charging stations with only a few charging points more frequently than locating only few charging stations with many charging points. Such design can be also favourable for the electricity network as it will not load the network largely at few locations, but the load is spatially more distributed.

- When we set the radius of charging points to $R_{\text{max}} = 1000$ meters, the number of charging opportunities gets high and the solved problem, especially during the busy weeks, becomes intractable when solved by a general purpose solver. This result indicates the limits of this methodology.

Although these initial results look promising, further steps are needed to refine the proposed approach. Considering the scheduling problem in the optimization model makes sure that there exists a time schedule to recharge all vehicles. However, this information is derived from the past data and it remains unclear how hard it is to find a feasible schedule in the system operation when the drivers do not have the prior information about the departure from the parking positions. Moreover, we assume that the parameter $R_{\text{max}}$ determines the maximum distance the drivers accept to drive from the parking position to the closest charging station. It could be beneficial to consider more complex strategies to determine the charging station the driver decides to use. Another challenge is in combining scenarios in a way that the resulting problem can still be solved for the long enough time period and we obtain as an output the robust design of the charging infrastructure that is suitable for all considered scenarios. To overcome the limits of proposed methodology, it would be beneficial to use heuristic approaches to expand the size of solved problems.

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**REFERENCES**


Paffumi, E., Gennaro, M. D., Martini, G., and Scholz, H. (2015). Assessment of the potential of electric vehi-


